A More Responsive M: Dike Injection via Diffuse Yielding in 2–D Rift Models

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Motivation

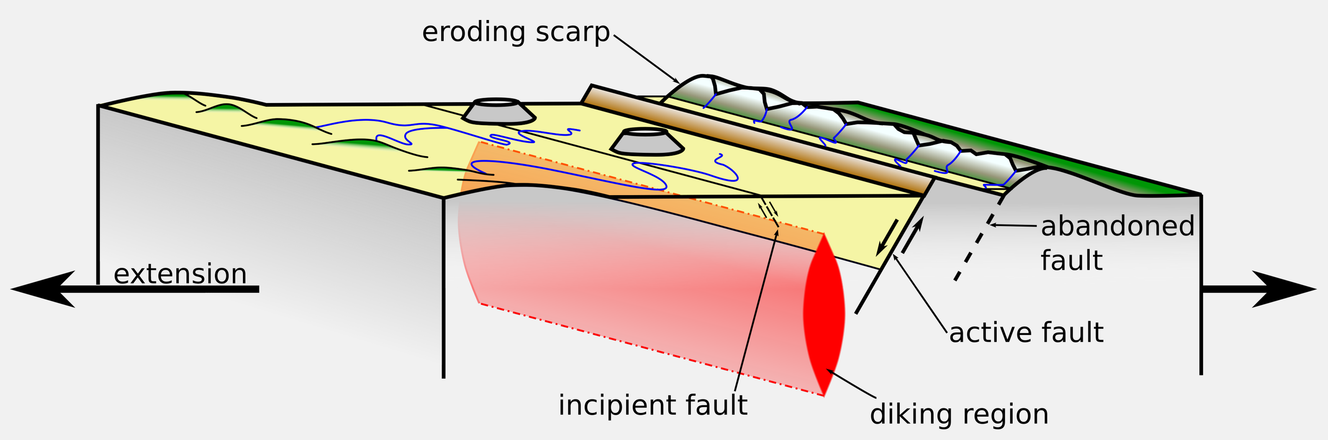


Figure 1: Conceptual model At continental rifts, extension is accommodated by faulting and magmatic intrusion (diking). During rift evolution, erosion and sedimentation continually alter the landscape, influencing the stress state of the lithosphere, the life-span of normal faults, and regions where magmatic injection may or may not be favorable.

- Magmatic accommodation of extension is often simulated in geodynamic models by imposing a divergence term in the continuity equation in a column of model elements. The ratio of the imposed divergence to total extension is the magmatic fraction of spreading, M.
- To date, most models impose a uniform M value through the domain and evaluate tectonic responses to different M values. Imposed uniform M values do not permit magmatic responses to changing lithosphere stresses, topography, and fault evolution.
- We present a computationally-inexpensive diffuse yielding formulation for M that responds to changing stresses, while still allowing for controlled variations in magmatic overpressure and supply.

Linking rift- and ridge- shaping processes (diking, faulting, erosion, and sedimentation) across timescales from hours to millions of years continues to present a challenge for long-term tectonics models. Here, we present a method for simulating magmatic injection in a rift or ridge environment that attempts to capture the dynamics of injection response to fault growth, topographic evolution, and evolving lithospheric stresses.

Numerical methods

Tectonic Model

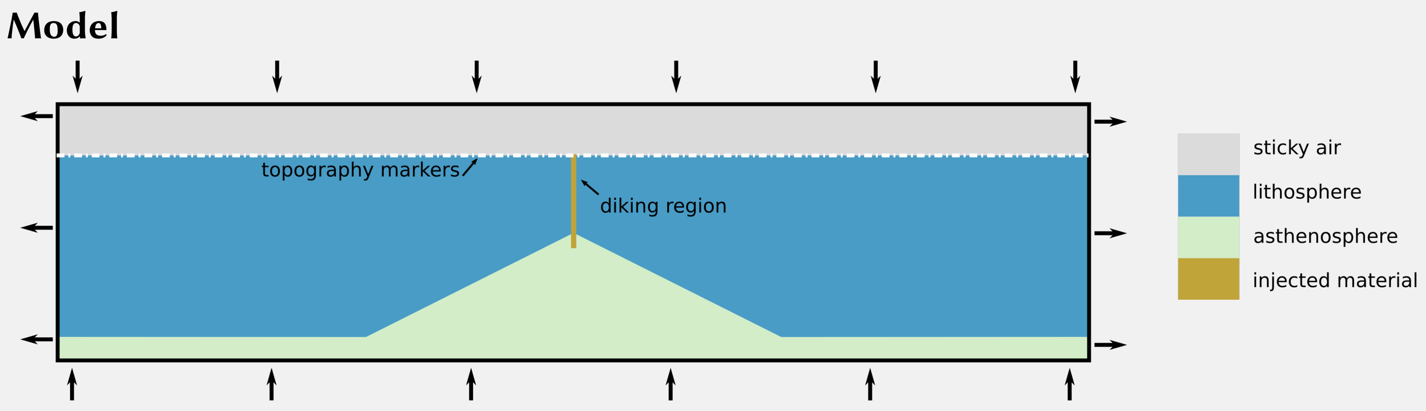


Figure 2: Model domain, boundary, rheology, and initial conditions

The numerical code **SiStER** [*Olive et al.,* 2016] solves for the conservation of mass (1), momentum (2), and energy(3) using a finite-differencing scheme on a fully staggered grid. In equations 1–2, v indicates velocity, σ'_{ij} denotes deviatoric stress (indices i and j indicate the vertical or horizontal direction respectively; repeated indices indicate summation), and T is temperature. Density, heat capacity, and thermal conductivity are ρ, c_p, and k, respectively. The right-hand side of equation 1 accounts for regions where dike injection occurs, where M is the fraction of magmatically accommodated extension, $U_{1/2}$ is half the extension rate, and dx is the dike injection width. In equation 3, $\frac{DT}{Dt}$ is the material time-derivative of T. We implement a visco-elastic-plastic rheology, assuming that the lithosphere behaves as a Maxwell solid; history terms for stored elastic stresses are added to the right side of Equation 2.

$$\frac{\partial v_i}{\partial x_i} = \begin{cases} \frac{2MU_{1/2}}{dx}, & \text{in injection region} \\ 0, & \text{otherwise} \end{cases} \qquad (1)$$

$$\frac{\partial \sigma'_{ij}}{\partial x_j} - \frac{\partial P}{\partial x_i} + \rho g_i = 0, \qquad (2)$$

$$\rho C_p \frac{DT}{Dt} = \frac{\partial}{\partial x_i}\left(k \frac{\partial T}{\partial x_i}\right) + H, \qquad (3)$$

Magmatic Accommodation of Extension

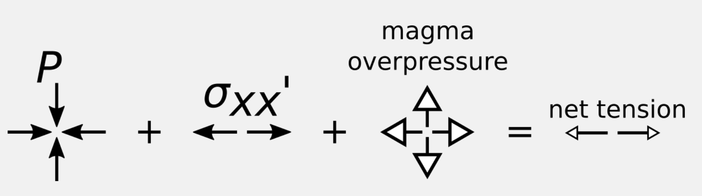
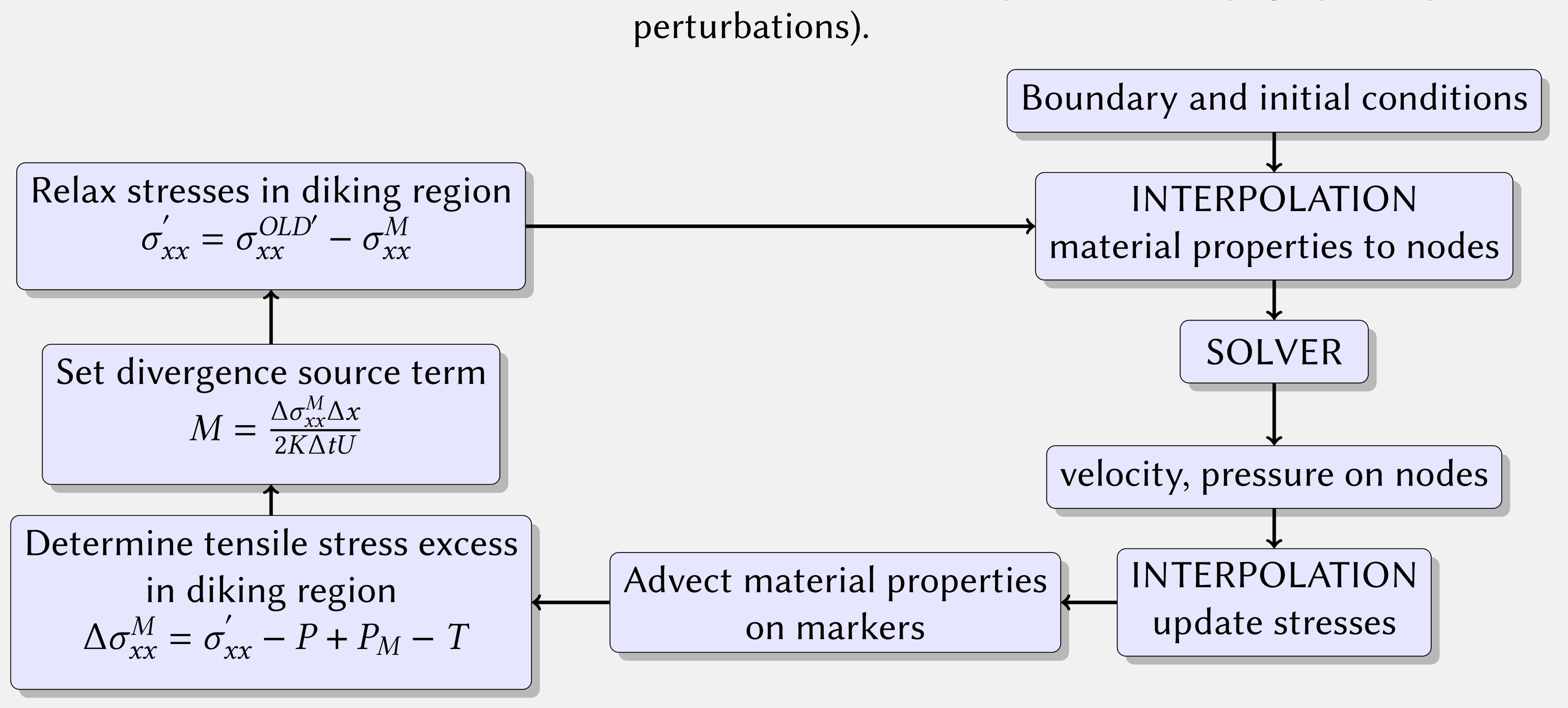


Figure 3: Tensile diking condition schematic

A region of diking and magmatically-accommodated extension is simulated using a modified approach from *Buck et al.,* [2005] and *Behn and Ito* [2008]. In this formulation, regions where magmatic injection accommodates extension behave as zones of diffuse yielding in the lithosphere. The yielding zone and amount of injected material responds to changes in stress conditions (e.g., interactions with nearby faults and topographically-driven stress perturbations).



At each timestep, we assess the net combination of deviatoric stress, pressure, and magmatic pressure in the diking region. If the net stress condition on a diking node is tensile, a volume of material is injected into the region via Equation 1 that corresponds to the tensile stress deficit.

$$\Delta\sigma^M_{xx} = \sigma'_{xx} - P + P_M - T \qquad (4)$$

$$M = \frac{\Delta\sigma^M_{xx}\Delta x}{2K\Delta t U_{1/2}} \qquad (5)$$

In Equations 4–5, $\Delta\sigma^M_{xx}$ is the tensile stress deficit in the diking nodes, determined as the sum of the deviatoric stress σ'_{xx}, pressure P, magmatic fluid pressure P_M, and a minimum tensile fracking strength T. M is the proportion of tectonic extension accommodated by magmatism, Δx is grid spacing, K is the magmatic elastic modulus, Δt is the tectonic time step.

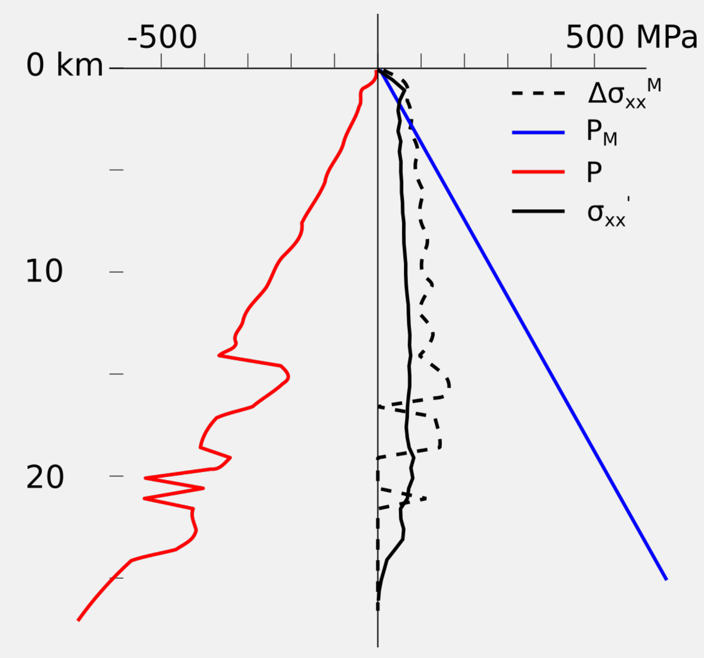


Figure 4: Example stress contributions to tensile diking determination (1 myr)

Magmatic Elasticity: A Proxy for Injection Frequency

The **magmatic elastic modulus**, K exerts an important control on M and overall model behavior. Lower K values result in a greater amount of injected material for a given tensile stress excess (see Eq. 5). K is serving a proxy for injection frequency between tectonic solves.

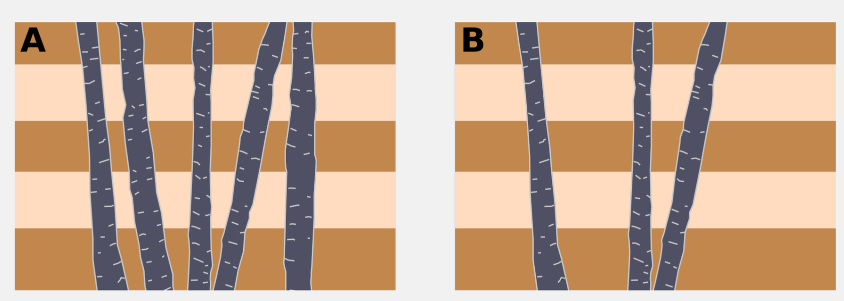


Figure 5: Comparative schematic for intrusion frequency with different K values

Suppose that in cases A and B, the tensile stress excess $\Delta\sigma^M_{xx}$ is equivalent. Over one tectonic time step, more material is injected in case A than in case B, as indicated by the dike density, such that

$$\Delta\sigma^M_{xxA} = \Delta\sigma^M_{xxB}$$
$$K_A < K_B$$
$$M_A > M_B$$

The diking formulation presented here produces normal faults in a simple 3-layer extensional model. The fault-bounded blocks behave similar to the scaling relationship presented in *Behn and Ito* [2008],

$$V_x = 2U(M - 0.5). \qquad (6)$$

The mismatch between experimental results and the scaling relationship (Eq. 6) may be a result of M variations in space and time, or the presence of two, simultaneously active faults.

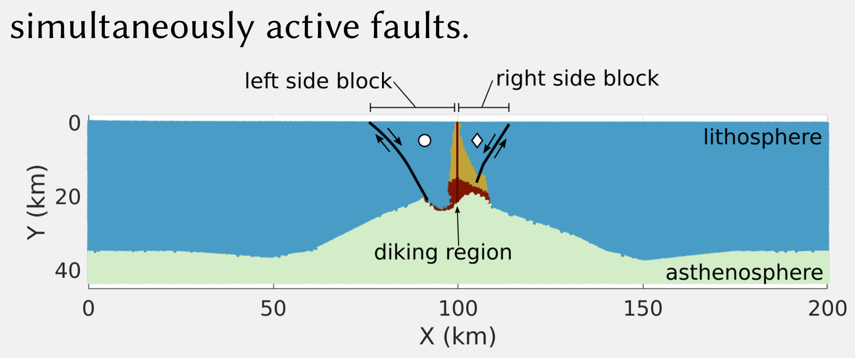


Figure 6: Simple three-layer tectonic model with an injection zone (orange/red) and two normal faults.

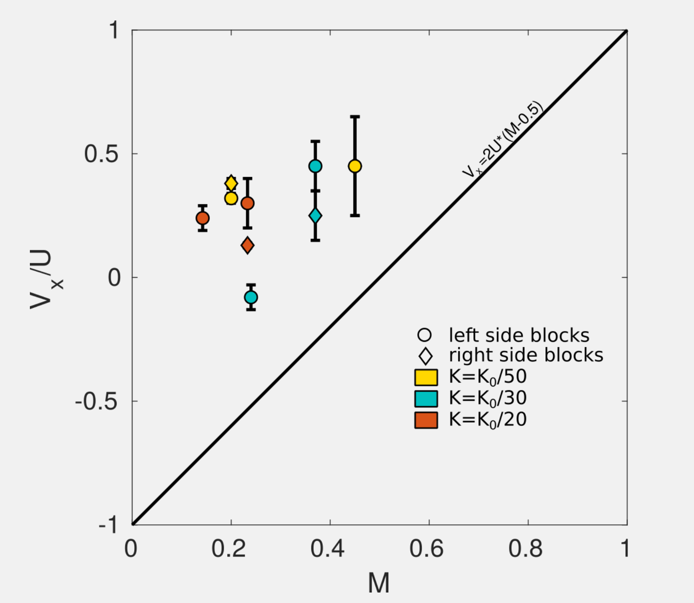


Figure 7: Comparison of numerical results with varying K values against *Behn and Ito's* [2008] scaling relationship.

Preliminary Applications in a Coupled Rift Model

Connecting topography, tectonics, and magmatic timescales

Preliminary results demonstrate complex interaction between surface processes, fault-generated topography, and dike injection during rift evolution. As rift flank topography grows, stress changes focus magmatic input to shallower depths.

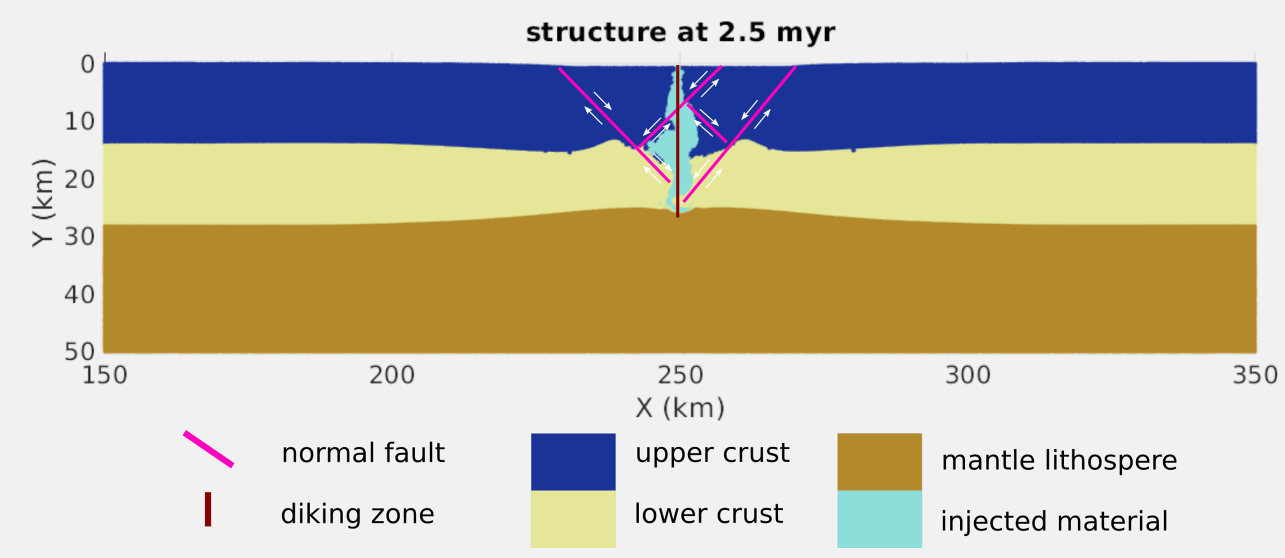


Figure 8: Model structure after 2.5 Myr of extension

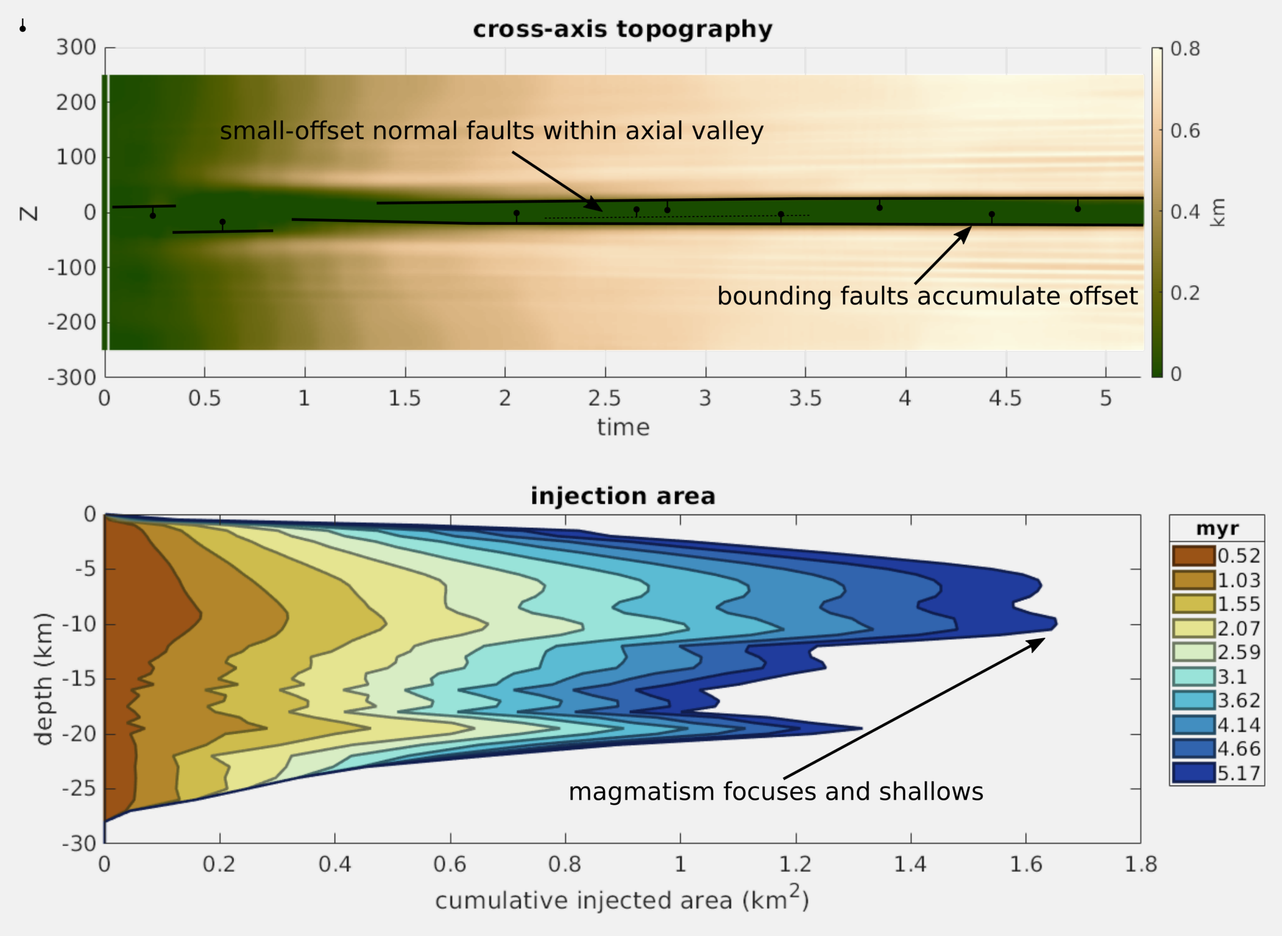


Figure 9: Temporal evolution of model surface and cumulative injection are shown with time.

For further discussion, join via Zoom on Thursday, July 30th at 4:00 pm EST: https://bccte.zoom.us/j/98587402574